THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and fifty-eighth regular meeting of the Society was held in New York City on Saturday, April 27, 1912. The attendance at the two sessions included the following fifty-two members:

Mr. J. W. Alexander, Professor R. C. Archibald, Mr. H. Bateman, Mr. A. A. Bennett, Professor W. J. Berry, Professor G. D. Birkhoff, Professor E. W. Brown, Professor B. H. Camp, Dr. Emily Coddington, Professor F. N. Cole, Dr. G. M. Conwell, Professor J. L. Coolidge, Professor E. S. Crawley, Dr. H. B. Curtis, Dr. L. S. Dederick, Dr. L. L. Dines, Mr. E. P. R. Duval, Professor H. B. Fine, Professor T. S. Fiske, Professor W. B. Fite, Professor O. E. Glenn, Mr. G. H. Graves, Professor C. C. Grove, Professor L. A. Howland, Mr. S. A. Joffe, Professor Edward Kasner, Professor C. J. Keyser, Dr. J. K. Lamond, Dr. N. J. Lennes, Professor W. R. Longley, Dr. H. F. MacNeish, Dr. Emilie N. Martin, Dr. E. J. Miles, Dr. H. H. Mitchell, Professor C. L. E. Moore, Dr. R. L. Moore, Professor W. F. Osgood, Mr. E. S. Palmer, Dr. Anna J. Pell, Professor A. D. Pitcher, Dr. H. W. Reddick, Professor R. G. D. Richardson, Dr. J. E. Rowe, Professor L. P. Siceloff, Mr. C. G. Simpson, Professor D. E. Smith, Professor Sarah E. Smith, Dr. W. M. Smith, Professor Henry Taber, Professor J. H. Tanner, Professor Oswald Veblen, Miss M. E. Wells.

The President of the Society, Professor H. B. Fine, occupied the chair. The Council announced the election of the following persons to membership in the Society: Miss S. R. Benedict, Smith College; Mr. C. E. Fisher, Rhode Island Normal School; Dr. T. H. Gronwall, Chicago, Ill.; Mr. L. A. Hopkins, University of Michigan; Dr. A. J. Kempner, University of Illinois; Mr. V. C. Poor, University of Michigan; Mr. R. B. Stone, Harvard University; Mr. K. P. Williams, Princeton University. Seven applications for membership in the Society were received.

It was decided to hold the annual meeting of the Society this year at Cleveland, Ohio, in affiliation with the American Association for the Advancement of Science. The winter

meeting of the Chicago Section will be merged in the annual meeting. Owing to President Fine's absence abroad, the delivery of his presidential address will be postponed to the annual meeting of 1913.

The following papers were read at the April meeting:

(1) Dr. R. L. MOORE: "Concerning Jordan curves in non-metrical analysis situs."

(2) Dr. J. K. LAMOND: "Improper multiple integrals over iterable fields."

(3) Professor L. A. HOWLAND: "Binary conditions for singular points on a cubic."

(4) Professor B. H. CAMP: "Certain integrals containing parameters."

(5) Dr. S. LEFSCHETZ: "On the V_3^3 with five nodes of the second species in S_4.

(6) Dr. E. R. MARSHALL: "A labor-saving device in computation."

(7) Professor G. D. BIRKHOFF: "The reducibility of maps."

(8) Professor G. D. BIRKHOFF: "A determinant formula for the number of ways of coloring any map."

(9) Professor OSWALD VEBLEN: "An application of modular equations in analysis situs."

(10) Dr. H. B. PHILLIPS and Professor C. L. E. MOORE: "A geometric use of matrices."

(11) Dr. H. B. PHILLIPS and Professor C. L. E. MOORE: "A theory of linear distance and angle."

(12) Professor L. P. SICELOFF: "Sylow subgroups in groups whose orders are of certain special forms."

(13) Professor A. D. PITCHER: "Concerning the continuity and convergence of functions of a general variable."

(14) Professor W. R. LONGLEY: "Proof of a theorem due to Picard."

(15) Mr. A. R. SCHWEITZER: "Remark on a functional equation."

(16) Mr. A. R. SCHWEITZER: "Theorems on functional equations."

(17) Dr. DUNHAM JACKSON: "On approximation by trigonometric sums and polynomials (second paper)."

(18) Dr. N. J. LENNES: "Concerning Van Vleck's non-measurable set."

(19) Dr. N. J. LENNES: "Concerning infinite polygons and polyhedrons."

The papers of Dr. Lefschetz, Professor Pitcher, Mr. Schweitzer, Dr. Jackson, and Dr. Lennes were read by title. Dr. Lefschetz's paper appeared in the May BULLETIN. That of Professor Longley is published in the present number of the BULLETIN. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. In the *Transactions* of this Society, volume 6, pages 83–98, Professor Veblen has given a definition of a simple plane curve lying in a plane which satisfies Axioms I–VIII, XI of his system of axioms for geometry. Let V denote such a plane. Dr. Moore wishes to show that:

I. There is a one to one correspondence between the points of V and a part H of an ordinary number plane, this correspondence being (in an easily defined sense) continuous. Thus each point of V is represented by a pair of coordinates (x, y).

II. A simple curve as defined by Veblen may be represented by a pair of equations $x = f_1(t)$, $y = f_2(t)$, where $f_1(t)$ and $f_2(t)$ are continuous functions of the numerical parameter t.

III. The plane V is an Ebene as defined by Hilbert in his article "Ueber die Grundlagen der Geometrie" (*Mathematische Annalen*, volume 56, page 383).

2. Dr. Lytle has shown that the fundamental relation

$$\underline{\int}_{\mathfrak{A}} f \leqq \underline{\int}_{\mathfrak{B}} \underline{\int}_{\mathfrak{C}} f \leqq \bar{\int}_{\mathfrak{B}} \bar{\int}_{\mathfrak{C}} f \leqq \bar{\int}_{\mathfrak{A}} f,$$

where the integrals are proper integrals, holds for a class of fields which he calls iterable. Using a new definition of an improper multiple integral, which is due to Professor Pierpont, and considering unlimited functions defined, in general, over iterable fields, Dr. Lamond gives conditions under which the above fundamental inequalities hold. In some cases the function f may have points of infinite discontinuity at an everywhere dense set.

3. The cubic plane curve has been exhaustively studied by use of the theory of ternary forms. Conditions necessary and sufficient for the existence of one or more double points, of a triple point, etc., have been derived in terms of ternary in-

variants, chiefly by Aronhold, Gordan, Clebsch, and Gundelfinger.

The theory of binary forms, however, may also be applied to the study of the cubic. Compare, for example, the treatment of the inflexions of a general cubic by Clebsch in his Theorie der binären Formen. In Professor Howland's paper conditions for the existence of a double point (node or cusp) and that of a triple point are derived, expressed in terms of the simultaneous invariants of two binary forms.

4. In a recent paper (*Annales de la Faculté de Toulouse*, series 3, volume 1 (1909), pages 25–128) Lebesgue has studied the integrals

$$\int_a^b f(t)\phi(t, n)dt, \quad \int_a^b f(t)\phi(t - x, n)dt.$$

His fundamental theorems have been generalized by Professor Camp so as to apply to functions of m variables and k parameters, defined over limited fields of very general natures. Moreover, corresponding theorems may be obtained for certain classes of functions not considered by Lebesgue. Applications of the results are found in the theory of the development of an arbitrary function in series of normal functions.

6. In insurance computations multiplications or divisions of a series of numbers by a constant are of frequent occurrence, in which the results are taken to the nearest unit, or 1st, 2d, or 3d, etc., decimal. A new method for obtaining the results of such serial multiplication or division with a minimum of labor is set forth in Dr. Marshall's paper. The method is based upon the following consideration:

Let $x_{0,1}, x_{0,2}, \cdots, x_{0,k}, \cdots; x_{1,1}, x_{1,2} \cdots, x_{1,k}, \cdots; \cdots, x_{i,1}, x_{i,2}, \cdots, x_{i,k}, \cdots; \cdots$ represent the required quotients, where the first subscripts indicate their nearest integers, and let $a_{0,1}, a_{0,2}, \cdots$ represent the corresponding variable dividend, while m is the constant divisor, so that $x_{i,k} = a_{i,k} \div m$. Then we have $x_{0,k} < .5 \leqq x_{1,k} < 1.5 \leqq x_{2,k} < 2.5 \cdots$.

Hence, in general, $a_{i,k} < (i + .5)m \leqq a_{i+1,k} < (i + 1.5)m$ (where the $=$ of the double symbol may hold only for $k = 1$). Hence all the values of $a_{i+1,k}$ giving the integer $i + 1$ lie between $(i + .5)m$ and $(i + 1.5)m$. For multiplication the

reciprocal of m, obtained to a certain degree of approximation, is used instead of the constant multiple m itself.

The author illustrates the process by a concrete example from insurance practice, and proves that on the arithmometer, which is an especially adapted calculating machine for this kind of computation, the saving of labor resulting from the use of this method is between 75 and 90 per cent. In the second part of his paper the author shows how to extend his method to the finding of quotients and products in which an approximation of a number of decimals is required and proves by actual illustration that his method can be advantageously used in a large field of actuarial practice, comprising divisions with moderately large differences of results, not exceeding two or three figures. It should be stated that the method is of most advantage only in cases where the differences of the results are smaller than those of the given series of multipliers or dividends. The illustrations of the second part of the paper are taken from Mr. George King's conversion and valuation tables appended to his paper, "On policies with deferred participation in profits, etc.," read before the Faculty of Actuaries in Scotland and printed in the *Transactions* of the Faculty, volume 5, part 9, number 83 (1911).

In conclusion the author shows how his method could also be used with considerable advantage in the computation of deferred and temporary annuities.

7. It is well-known that the problem of coloring a given map in four colors is reducible to that of coloring other maps of fewer regions in case the given map has (1) any multiply connected regions; (2) more than three regions meeting at any point; (3) any region bounded by less than five others; these facts comprise nearly all that has been done toward the reduction of maps.

By defining reducibility in a technical sense in which a principle due to Kempe is involved, Professor Birkhoff shows that numerous other configurations occur only in reducible maps; thus any ring of less than five regions and a ring of five regions not bounding a single region are proved to occur only in reducible maps.

8. This note by Professor Birkhoff on a formula for the

number of ways of coloring a map in any number of colors will appear in the *Annals of Mathematics*.

9. In the note by Professor Veblen it is shown that the circuits and systems of circuits in a complex of lines (in the sense of analysis situs) may be represented by the solutions of a system of linear equations, modulo two. The operations of linearly combining these equations or their solutions are interpreted geometrically. A similar theory holds for complexes of n dimensions. The equations may be applied in various problems of analysis situs, including the four color problem and the proof of Euler's theorem.

10. A point A in n dimensions is determined by $n+1$ homogeneous coordinates and may be represented by the matrix $\|a_1 \cdots a_{n+1}\|$. A linear function $\lambda A + \mu B$ of two points is represented by the one-rowed matrix whose elements are $\lambda a_i + \mu b_i$. A line AB is represented by a two-rowed matrix $[AB]$ formed by writing the coordinates of A above the coordinates of B. The elements of this matrix are the two-rowed determinants obtained from it and not the individual coordinates. The sum of two such matrices is formed by adding the corresponding elements. This new matrix is not, in general, expressible as a two-rowed matrix of $n+1$ columns. Similarly, planes are represented by three-rowed matrices $[ABC]$, three-spaces by four-rowed, etc.

These quantities $[AB]$, $[ABC]$, etc., may be considered as products obeying the laws of Grassmann's outer product, and are called progressive. The operation consists of writing the coordinates of the first factor in the first row, the coordinates of the second factor in the second row, etc., to form a new matrix.

Hyperplanes are used as coordinates in the same way and the products thus obtained are called regressive. If the number of points representing the factors of a product is greater than $n+1$, the factors are expressed in plane coordinates and multiplied regressively. One of the chief aims of Dr. Phillips and Professor Moore is to express this regressive product in terms of points, thus obtaining Grassmann's reduction formula.

11. In this paper Dr. Phillips and Professor Moore define

the distance between two points, in three dimensions, by the following properties: (1) If one point is fixed and the distance held constant, the other describes a plane. (2) Distances between points on a line have the additive property. It is found that the distance is expressible in terms of a fundamental system consisting of a plane and a linear complex. Distance will then be invariant under a six-parameter group of collineations leaving the fundamental system fixed. The angle between two planes is defined as the dual of the distance between two points. In this case the fundamental system consists of a point and linear complex. In order that distances and angles be left invariant by the same six-parameter group of collineations, the complexes are taken as the same and the point as the pole of the plane with respect to the complex. In terms of this fundamental system angle between lines, distance from point to line, etc., are defined. Formulas for the area of a triangle and volume of a tetrahedron are also obtained. These results are generalized for n dimensions.

12. In this paper Professor Siceloff studies restrictions on the structure of a group corresponding to certain restrictions on the form of its order. He proves in the first theorem that a group of order $(kp + 1)p$ cannot have the full number, $kp + 1$, of subgroups of order p if p is greater than some prime-power factor of $kp + 1$. The second theorem states that in a group whose order has only one factor (other than unity) $\equiv 1 (\mathrm{mod}\ p)$, the elements common to two Sylow subgroups of order p^a form an invariant subgroup. The third theorem deals with an important special case under the hypothesis of theorem 2.

13. In his Introduction to a Form of General Analysis and in his lectures at the University of Chicago Professor E. H. Moore has given a theory of developments of a general range and has indicated the beginning of a general theory of continuity and convergence of functions relative to a development of the range of the functions. Professor Pitcher secures the ordinary properties of continuous and convergent functions by means of simple postulates on the development of the range of the functions.

15. The purpose of Mr. Schweitzer's first paper is to explain

a reference to *Crelle's Journal* given on page 300 of his paper, "On a functional equation," BULLETIN, volume 18, pages 299–302. The relation $\phi(x,y) = \Omega\{\psi(x) - \psi(y)\}$ can be written $\Omega^{-1}\{\phi(x,y)\} = \psi(x) - \psi(y)$, and the preceding reference suggests $\Omega^{-1}\{x\phi(y) - y\phi(x)\} = \psi(x) - \psi(y)$ as a correlative of the equation of Abel, *Crelle*, volume 2, page 389. Corresponding to Abel, l. c., page 386, one gets a particular case of the relation $\chi\{\phi(x,y)\} = \chi(x) - \chi(y)$ by assuming that $\Omega^{-1}(x) = \psi(x)$ in the preceding correlative. It is of interest to note here that C. Lottner, *Crelle*, volume 46, pages 367, 368, etc., has discussed a special case of $\Omega^{-1}\{\phi(x,y)\} = \psi(x) + \psi(y)$ in relation to the preceding paper of Abel.

16. In connection with his recent article in the BULLETIN, March, 1912, page 300, Mr. Schweitzer proves the following theorems:

I. If $\phi[x, \phi(y, z)] = \phi[y, \phi(x, z)]$ and $\phi[\phi(y, z), z] = y$ and $\phi[\phi(y, z), y] = z$, then there exists a function χ_ϕ such that $\phi(x,y) = \chi_\phi^{-1}\{\chi_\phi(x) + \chi_\phi(y)\}$ and $\phi(x,y) = \chi_\phi^{-1}\{\chi_\phi(x) - \chi_\phi(y)\}$.

II. If $\phi[y, \phi(x, z)] = \phi[z, \phi(x, y)]$ and $\phi[\phi(y, z), z] = y$ and $\phi[\phi(y, z), y] = z$, then there exists a function χ_ϕ such that $\phi(x, y) = \chi_\phi^{-1}\{\chi_\phi(x) - \chi_\phi(y)\}$ and $\phi(x,y) = \chi_\phi^{-1}\{\chi_\phi(x) + \chi_\phi(y)\}$.

17. Following out further the line of investigation on which he reported at the February meeting, Dr. Jackson is led to the following conclusions: If $f(x)$ is a function of period 2π which satisfies a Lipschitz condition with coefficient λ, that is, if it is always true that

$$|f(x_2) - f(x_1)| \leqq \lambda |x_2 - x_1|,$$

then $f(x)$ can be approximately represented for all values of x by a trigonometric sum of order n or lower with a maximum error not exceeding $3\lambda/n$, for all positive integral values of n. If $f'(x)$ satisfies a Lipschitz condition with coefficient λ, the maximum error of such an approximation for $f(x)$ can be made not to exceed $20\lambda/n^2$. If $f(x)$ satisfies the same conditions, except as to periodicity, in a closed interval of length 1, it may be represented in this interval by a polynomial of the nth or lower degree with an error not greater than $\frac{3}{2}\lambda/n$ or $10\lambda/n^2$ respectively. If $f(x)$ satisfies the first of the conditions above, it differs from the partial sum of its Fourier series, to terms of the nth order, by not more than $6\lambda \log n/n$, provided $n \geqq 5$.

18. In this paper Dr. Lennes describes in detail the construction of a set which is non-measurable in the sense of Lebesgue. The general method is the same as that used by Van Vleck (*Transactions*, volume 9, page 237), but each step is made to depend upon explicitly formulated postulates.

19. In two papers published in the *American Journal of Mathematics*, volume 33, Dr. Lennes formulated a certain body of theorems on polygons and polyhedrons. With one exception these were confined to figures having a finite number of sides or faces. In the present paper many of the results obtained in these papers are extended to polygons and polyhedrons having an infinite number of sides or faces.

F. N. COLE,
Secretary.

PROOF OF A THEOREM DUE TO PICARD.

BY PROFESSOR W. R. LONGLEY.

(Read before the American Mathematical Society, April 27, 1912.)

CONSIDER the ordinary differential equation of the first order and second degree

$$(1) \qquad Ap^2 + 2Bp + C = 0 \qquad (p = dy/dx),$$

in which the coefficients are power series in x and y vanishing when $x = y = 0$

$$(2) \qquad A = ax + a_1y + \cdots, \quad B = bx + b_1y + \cdots, \quad C = cx + c_1y + \cdots.$$

Picard has proved that in the general case* every integral curve (real curve in the cartesian plane) of equation (1) which comes infinitely near the origin, actually reaches the origin with a determinate tangent. The proof† is based upon the existence of an analytic integral curve passing through the origin, a condition which is satisfied in the general case. But such a curve does not always exist, and the following proof, which

* By the general case it is meant that there exists no particular relation of equality among the coefficients of (2).

† Picard, *Comptes Rendus*, vol. 120 (1895) p. 524; *Math. Annalen*, vol. 46 (1895), p. 521; Traité d'Analyse, vol. 3, pp. 217–225.

makes no use of its existence, gives more information concerning the form of the integral curves near the origin.

To examine the behavior of integral curves in the neighborhood of the origin we may limit the coefficients in equation (1) to terms of lowest degree and we suppose that A, B, and C contain only first degree terms in x and y. Solving equation (1) for p gives

$$p = \frac{-B + \sqrt{\Delta}}{A}, \tag{3}$$

where $\Delta = B^2 - AC$. Through every point at which Δ does not vanish there are two integral curves with distinct tangents depending on the choice of the sign of the radical. Suppose this choice has been made. Then the form of equation (3) shows that the slope depends only on the ratio y/x and hence the slope of an integral curve crossing the line $y = mx$ is the same at every point of the line. From this it follows that it is possible to trace a continuous integral curve by making the radius vector from the origin turn always in the same direction, provided the curve does not reach a point at which $\Delta = 0$ where the sign of the radical could be changed.

The form of the integral curves depends on the nature of the discriminant locus, the equation of which is $\Delta = 0$, and there are two cases to be considered: (1) when the discriminant locus has a conjugate point at the origin, and (2) when the discriminant locus consists of two distinct branches through the origin.

In the first case Δ vanishes for $x = y = 0$ and is positive for all other values of x and y. Hence if an integral curve does not pass through the origin it may be followed by making the radius vector rotate always in the same sense about the origin. It is conceivable that an integral curve might have the form of a spiral approaching the origin asymptotically. To do this the slope must change sign four times during one revolution of the radius vector; twice by passing through zero (at the highest and lowest points) and twice by passing through infinity (at the extreme right and left points). To see that this is impossible we consider equation (3). The lines $A(ax + a_1y = 0)$, $B(bx + b_1y = 0)$, and $C(cx + c_1y = 0)$ pass through the origin and by a rotation of the axes the line B may be taken for the X-axis. The line C is the locus of points where the slope of integral curves is zero. Starting at any

point near the origin on the positive X-axis with a definite sign for the radical we may follow an integral curve by making the radius vector rotate always in a counterclockwise direction. If the slope changes sign by passing through zero when the curve crosses the line C above the X-axis, then p will not change sign when the same line is crossed below the X-axis, because the sign of the radical remains the same while the sign of the quantity B is reversed. Hence the curve can not meet the positive X-axis a second time and the spiral is impossible.

In the second case the discriminant locus has two distinct branches passing through the origin. In one of the angles between the branches Δ is positive and there exist real integral curves, while in the other angle Δ is negative and there are no real integral curves. By a transformation of coordinates we may take the two branches of the discriminant locus for the coordinate axes so that Δ is positive in the first and third quadrants and negative in the second and fourth quadrants. Since $\Delta = B^2 - AC$ is positive in the first quadrant and negative in the second it is evident that the lines A and C must pass through the origin with positive slope, that is, they must lie in the first quadrant. In this case an integral curve can not have the form of a spiral, but it is conceivable that it might oscillate between the coordinate axes an infinite number of times approaching the origin. To see that this is impossible suppose that we choose a definite sign for the radical in equation (3) and begin to follow an integral curve from a point on the positive X-axis. The radius vector must rotate in a counterclockwise direction until the Y-axis is reached. To follow the curve further the sign of the radical must be changed at this point and the radius vector turned clockwise towards the X-axis. Each time the curve traverses the first quadrant it must cross the line A once and the line C once. In crossing the line C with one sign for the radical the slope of the curve will change sign by passing through zero, while with the opposite sign for the radical the slope of the curve will not change sign in crossing C. Similarly in crossing the line A with one sign for the radical the slope will change sign by passing through infinity, while with the opposite sign for the radical the slope will not change sign. Hence in starting from the X-axis, crossing to the Y-axis, and returning to the X-axis the slope of an integral curve must change sign twice and only

twice; once by passing through zero (at a maximum or minimum point) and once by passing through infinity (at an extreme right or extreme left point).

Suppose now an integral curve Γ leaves the X-axis with positive slope. In order to reach the Y-axis the slope must become negative when Γ crosses A. Then in crossing C (either the first or second time) the slope will change to positive by passing through zero, and with a positive slope Γ can not return to the X-axis.

Suppose Γ leaves the X-axis with negative slope and meets A first. In order to reach the Y-axis it must cross A the first time without changing to positive slope. After having crossed C twice the curve will return towards A with positive slope and can not again meet A because at the second point of intersection the slope of Γ would have to be infinite. Hence Γ can not return to the X-axis.

Suppose Γ leaves the X-axis with negative slope and meets C first. (a) If the slope does not change sign then Γ, after crossing A twice, will return towards C with positive slope and can not meet C again because at a second point of intersection the slope would have to be zero. (b) If the slope changes to positive at the first intersection with C then the curve could not cross A twice since the slope would have to be infinite at one of the points of intersection. Hence Γ can not return to the X-axis.

The preceding argument establishes the theorem as given by Picard. No mention has been made of the case in which the discriminant locus has a cusp at the origin. If the coefficients in equation (1) are limited to terms of the first degree, a cusp at the origin means that Δ is a perfect square and the equation can be broken up into linear factors. In this case the spiral is a possible form for integral curves, but this conclusion does not follow when the higher degree terms are present unless the cusp degenerates into two coincident branches.

SHEFFIELD SCIENTIFIC SCHOOL,
NEW HAVEN, CONNECTICUT.

A TREATISE ON PHYSICS.

Traité de Physique. Par O. D. CHWOLSON. *Ouvrage traduit sur les éditions russe et allemande* par E. DAVAUX. *Edition revue et considérablement augmentée* par l'Auteur, *suivie de Notes sur la Physique théorique* par E. et F. COSSERAT. Tome 1: *Mécanique. Etats de la matière. Acoustique.* vii + 1092 pp. 1908. Tome 2: *L'Energie rayonnante.* 1188 pp. 1909. Paris, A. Hermann.

DESPITE the fact, and perhaps for the very reason that teachers of the first general collegiate course in physics, which usually falls in the sophomore year, are widely and chronically of the opinion that there exists no text suited to their needs, there are upon the market a great variety of elementary general treatises on physics, varying from a moderate volume of three or four hundred pages to lengthy works of three or four times the size, like that old standby Ganot. When we come to more advanced compendia, suitable for the maturer student of physics, we have not many to choose from, and none, so far as we recall, in the English language. Indeed it might well be questioned whether, with the present extended and rapidly growing domain of physics, it would be possible for a single author properly to cover the whole ground, whether it might not be necessary to fall back upon collaborations as in Winkelmann's Handbuch der Physik. We may remember that over ten years ago A. Gray printed a first volume of a Treatise on Physics,* which in its masterly manner gave promise of being just what was needed in this line; but apparently the work has been abandoned, much to our disappointment. However, Chwolson has succeeded in carrying a similar task, monumental as it is, to an eminently satisfactory conclusion.

So long as Chwolson's work remained in the original Russian it could be of little use outside of Russia. But as surely as a first class work appears the Germans, often in the guise of the house of B. G. Teubner, are apt to find it out and to publish a translation; this time, however, it was the firm of Vieweg und Sohn who rendered the German and international public the service of printing Chwolson's treatise in German. That

* Reviewed in this BULLETIN, volume 8, p. 403.

Hermann should continue the international conquest of the work is testimony not only to its high value, but to his own courage; for the French seem none too prone to take kindly to foreign texts, and the undertaking of a four volume treatise on physics, to run into thousands of pages and to sell at about 150 francs, is no light tax on one's faith and one's resources. It is with great pleasure that we observe that the demand for the work has been so great as already to necessitate a second edition, at least of the earlier fascicules.* As the French edition is the latest, it is also the best; for the author has been able to revise and elaborate. The translator has added some notes, and E. and F. Cosserat have furnished additional contributions.

The work commences with a general introduction which shows that Chwolson is somewhat of a philosopher. It is probably true that everybody who has large views on a wide range of phenomena must have some sort of philosophy which is interesting to him to expound and interesting to others to read, but which in the last analysis is rather a personal matter with which almost everybody else may differ. For instance the author states that the more limited the number of hypotheses, the more advanced the state of science. This is a beautiful ideal that appears to have little realisation in practice. From the opposite point of view one might just as well hold that the more advanced the state of science, the greater the diversity of hypotheses, and the better known the limitations or field of application of each. The hypothesis of imponderable caloric has never disappeared, and probably never will, from the theory of the flow of heat. To be compelled to treat heat as molecular motion in this case would be entirely too inconvenient. Some physicists now desire to abolish the hypothesis of an ether. They cannot do it; the best they can do is to formulate a new hypothesis to organize phenomena which are not satisfactorily organized under the ether-hypothesis. But as our philosophy cannot be as valuable to our readers as the author's should be to his, we should be lacking in humor if we persisted.

An elaborate discussion of physical quantities or magnitudes of different sorts and of units (not the theory of dimen-

* What was tome 1, fasc. 1, of 407 pages in 1906 has now become in 1911 tome 1, vol. 1, considerably larger. (I am reviewing from the earlier edition, which has only recently come to me for that purpose. E. B. W.)

sions) is a valuable contribution to the introduction. This is a subject which gives many students considerable trouble, but should not worry readers of this work. The notion of density, for example, is used in very different ways in physics, and needs the careful treatment the author gives. We must observe, however, that for all his care he defines ordinary density as it is ordinarily defined, namely, as a limit, whereas in reality on the molecular hypothesis no such limit can exist. This is but another instance of the diversity of hypothesis; no one disputes the molecular view of matter, yet few take the precaution to observe that the contradictory hypothesis of a continuum is constantly being used. We have no objection to this plurality of hypotheses, but we would not slur it over.

The exposition of the relative fields of experimental, theoretical, and mathematical physics is lucid. It is doubtful, however, whether the mathematical method does not encroach more upon and is not more necessary to theoretical physics than seems to be implied on page 11; and indeed the work as a whole seems to give more space to mathematical physics* than one might imagine from the introductory statements. It should be pointed out that mathematics is used in physics not merely to deduce consequences from well established facts, but to formulate new hypotheses and enquire after their consequences. In this latter sense much mathematical physics is as truly experimental as is considerable laboratory work.

The introduction closes with some mathematical work on differentiation, curvature, and vector addition. This is necessarily very meager and will be sufficient only for the more elementary parts of the text, some portions of which require very considerable mathematical training and ability. As references to original sources are very numerous throughout the remainder of the work, and form a very valuable part of it, the author inserts here a list of the most important periodicals cited, with their appropriate abbreviations.

Some elementary mechanics comes next—a little kinematics, rectilinear and curvilinear, the analysis of inertia, mass, and force according to Newton's laws, momentum, impulse, resultants, couples, center of inertia, and moment of inertia. The principle of the conservation of mass is not mentioned without adding a list of references to various recent experi-

* Due mainly to additions by translator and editors, and unfortunately of the French non-physical type.

ments attempting to detect changes (not, of course, the very recent experiments in connection with moving charges and the principle of relativity). The author states that there is a very general notion that Newton's second law refers to rate of change of momentum, and that he cannot share that opinion, but regards the law as dealing with acceleration. This same point of view is taken by Jouguet in his searching historical study of mechanics.* The English attitude, however, has seemed to incline more to the rate of change of momentum; and it may be interesting to note that it was by going back to the definition of force as rate of change of momentum that G. N. Lewis was able in studying relativity to assign a definite mass to a particle, instead of the varying masses from transverse to longitudinal that previous writers had used.

The author refers for the determination of centers of inertia to earlier and more elementary studies of the reader; in finding moments of inertia he uses triple integration without apology, except to say that readers unacquainted with the method may take the results for granted. When he goes on to treat work and energy, he uses sums, limits of sums, and simple integrals, even line integrals, without hesitation. We cannot help wondering how necessary it is to develop the small amount of differential calculus above mentioned if one is to go right on with integral signs. It seems as though it might have been better either to give a considerable account of mathematical analysis or to have taken the whole of elementary calculus for granted. In studying energy, the various forms (not merely the kinetic and potential energies of mechanics) are taken up and the three principles, namely, that the energy is a determinate function of the state, that energy is conserved, and that it may possess a direction of transformation, are elaborated.

The treatment of harmonic motion is detailed, including the composition of vibrations, damped oscillations, propagation of a disturbance by longitudinal or transverse waves, wave surface and ray, Huyghens's principle, reflexion, refraction, diffraction, and Doppler's principle. As the material covers over 60 pages, it is readily seen to be sufficiently emphasized. The author next proceeds to study universal gravitation and the newtonian potential with the minimum amount of mathematics, parabolic and pendulum motion in the field of gravity,

* E. Jouguet, Lectures de Mécanique, reviewed in this BULLETIN, vol. 18, p. 32, without mentioning this particular point.

and the theory of dimensions as applied to mechanics. There seems to be no need of any other comments than these: The treatment of the ordinary mechanics of particles and rigid bodies seems very meager and the use of mathematics extremely limited for a work of this size, which itself subsequently gives considerable development to the mechanics of continuous media. In this particular, the work appears unbalanced and inferior to Gray's Treatise on physics. We should recommend the addition of some theoretical mechanics with Gray as a model, although it would not be necessary to carry the matter so far as he does. The bibliography on mechanics is extensive and well chosen; but in regard to this list, and most others in the course of the work, we have been wholly unable to see why the author or editor did not make the arrangement according to date or alphabetical according to author or title instead of tossing the references together apparently pell-mell or following the order in which they were cited in the text.

We have said that as far as mechanics is concerned Chwolson's work is unbalanced; but we will hasten to add that this may not be due so much to his own doings as to his polite complacency in accepting the additions of others. For right here (pages 236–273) we come upon a long note by the Cosserats upon the dynamics of a point and of a rigid body, which has absolutely no place in this work, and should have had much less space even if the author had developed his mechanics to include Lagrange's equations and Hamilton's principle. We postpone further comment.

The following 125 pages are given to instruments and methods of measurement. To one not at all an experimental physicist this makes clear, pleasant, and instructive reading. There is good general advice as to methods of measurement, the elements of least squares, formulas of approximation, numerous diagrams of instruments designed to make mechanical (as distinguished from thermal or electric) measures, and ample explanation. Even the experiments to determine the mean density of the earth are discussed. (Why Burgess is not mentioned in this connection we do not understand.) There is a very appropriate note on the theory of integrating machines by the translator.

The second fascicule of Volume I deals with the gaseous state. If we are not mistaken, the author's choice of hydrogen (H = 1) as standard instead of oxygen (O = 16) is now some-

what old-fashioned. The experiments necessary for testing the equation $pv = RT$, and the results obtained, are naturally the first. In future editions of the book the work of P. W. Bridgman, who has succeeded in running pressures from 3,000 up to 12,000 atmospheres, will have to be cited with Natterer's, Cailletet's, and Amagat's on page 429. Clapeyron's formula and van der Waals's equation, with some of the modifications that have been suggested, are merely touched at this point. There follows a chapter on barometers, airpumps, and similar apparatus.

After some mention of the conduct of gases in the presence of other gases or liquids or solids (laws of Dalton, Henry, etc.), Chwolson takes up the kinetic theory and, while not going into very complicated mathematical analysis, is quite able to show the relations of the theory to the previously developed laws, and to discuss the mean velocity, mean free path, viscosity (Maxwell's law), and van der Waals's equation. Sutherland's formula for the viscosity is given without going into his theoretical deduction. Indeed it is characteristic of the author's good sense and balance to run through the development of such an important theory as the kinetic, and then add without any theoretical derivation some formula which has been found to fit the experimental data better than the one derived from that theory. The interesting chapter on the motion of gases (efflux, diffusion, resistance to projectiles, dissociation, etc.) is rendered somewhat unintelligible in our copy by the fact that pages 537–552 are omitted and 521–536 repeated through an error in binding, but enough can be seen to show that the interesting work of Hugoniot and recent formulas by Boussinesq are not overlooked.

A fascicule of over 300 pages is given to the properties of the liquid and solid state. For the most part the presentation is descriptive rather than mathematical or theoretical. Surface tension and capillarity receive considerable attention. The theory of electrolytic dissociation is mentioned, but the facts about solutions are here sketched without much attention to that theory, which is reserved for a later volume. In regard to osmotic pressure the author likewise refrains from adopting definitely any hypothetical explanation, contenting himself with emphasizing the difficulty of finding a satisfactory explanation by calling attention to several of those which have been suggested. Friction and viscosity are briefly discussed.

The treatment of the motion of liquids is partly descriptive, as might be found in a work on hydraulics, and partly mathematical, due perhaps to various additions by the translator. This makes the chapter read somewhat unevenly. Indeed it seems almost useless to include even the small amount of the theory of vortices here given unless the reader has had more thorough training in the composition of angular velocities than was suggested in the early work on mechanics. Moreover it is rather unfortunate, from the molecular point of view, to use such an expression as "the two molecules are always upon a vortex line" (page 711). It may be wondered whether this is Chwolson's own language.

The chapter on the colloidal state has evidently been written or at least edited by V. Henri, which is sufficient recommendation for it.

In regard to the treatment of the solid state there seems to be no need of comment. The same unevenness between the extremes of mere description on the one hand and considerable difficult mathematics of elasticity on the other is noticeable; indeed there are parts of the latter which would still seem of rather doubtful utility in this work even if the more extended treatment which we should recommend for mechanics were given.

In a great treatise on physics, like this, mechanics, hydromechanics, and elasticity should be introduced just so far as they develop and illustrate fundamental physical principles either in themselves or in their applications. The ideas of momentum, moment of momentum, and energy cannot be too highly emphasized, especially in view of Maxwell's theory of electromagnetics and modern theories on radiant energy. We should introduce the gyroscope (not here mentioned, so far as we observe*) because there is a lot of physics in it that can hardly be so well found elsewhere. We should discuss Lagrange's coordinates and Hamilton's principle because without these it is difficult to get the idea of a general dynamical system and of the most important single principle in mathematical (may we say theoretical?) physics. We should study hydrodynamics and elasticity for the physical principles involved, for the conceptions connected with dynamics in a medium, to emphasize the analogies and points of difference between liquid and solid media, the ether, and discrete systems.

* Except in connection with the gyrostatic theory of light!

We should bring to light and use general principles of reciprocity, and the like. But we should be rather wary of introducing mathematical developments which are devoid of general physical significance even though they lead to formulas of engineering or laboratory practice—unless we are frankly intent on including a good account of mathematical (as distinguished from theoretical) physics. It would not do to say that in the present edition of Chwolson's Physics there is too much mathematical work; but it is true that the physical side of a question seems almost immediately lost forever as soon as the analysis begins. This is a difficulty that is almost always in evidence in a French, as contrasted with an English, text; and if we rightly detect the uniformly physical spirit in which Chwolson himself writes, we cannot help feeling that the unevenness and mathematicity are not his.

These remarks apply also to a number of sections in the final fascicule (Acoustics) of the volume. What propriety is there in the sectional heading "Le problème général de l'acoustique. Conditions aux limites" in a work whose author in his introduction states that it is theoretical, not mathematical, physics that is important? It is hard to believe that boundary value problems really interest him. Still, although we do not feel that the translator and editors have made the best selection of additional mathematical material or have incorporated it in the smoothest fashion, we do not wish to give the impression that there may not be many readers of the work who will find these additions helpful and some who may find them the most helpful part of the treatise. There will be all sorts of readers with every diversity of requirement; and the editors probably had some particular class of readers in mind. Moreover, we have a feeling, and perhaps the translator had it, that purely mathematical physics is so widespread that some concession to it should be made. The average density of mathematics in the treatment of sound appears greater than in previous fascicules.

Although the usual topics are taken up, the author has space for a number of things not ordinarily mentioned, the propagation of finite displacements, explosive waves, where the velocities exceed the canonical. On the other hand, although diffraction of sound and the question of hearing are both treated, we do not find any mention of the way in which we perceive the direction from which the sound comes. It is

interesting to note that in the section on longitudinal vibrations of strings the references are to the recent (1906) work of H. N. Davis. It is sometimes surprising how thoroughly the author has covered the literature up to the date of printing; it is by no means infrequent to see references dated within two years of the issue of the fascicule in which they are quoted. Apparently, however, the ten year old work of E. W. Scripture,* which would interest some students of sound, is not cited.

Volume I ends without an index; so does the next volume. If the complete work carries no index, or even if it carries only a poor one, a large amount of the usefulness of the treatise will be lost.

The second volume has the title Radiant Energy—a brave title. And the first fascicule displays a subheading Emission and absorption of radiant energy—a stalwart start. The complete adoption of the Maxwell theory, the firm belief in the reality of the ether and in its importance in the explanation of nature, the relentless grounding of the student in the use of a medium are characteristics of Chwolson's work which are more in evidence than in most other books and which rank his as a distinctly modern treatise.

And yet we of 1912 must admit that we cannot use the titles Radiant energy and Emission and absorption of radiant energy with the whole hearted confidence that we could wish, or that we could fairly assume five, or at any rate ten, years ago. We are in the unfortunate transitional stage of knowing too much and yet too little about radiant energy, its absorption, and its emission. We know too much to make the facts fit the Maxwell theory as we today understand it; we know too little or have too little genius as yet to found a new theory. There is a vast array of evidence that radiant energy appears usually, if not always, in discrete particles; there is a tendency to abandon the ether.

Chwolson should not be blamed for not taking toward the ether that attitude which has not yet ripened thoroughly and which was scarcely budding when the translation of his book began; rather should he be blamed if he had adopted it in place of the straight Maxwell theory, which even yet is none too clearly appreciated by the students to whom the author is talking. We cannot, however, withhold an adverse comment

* Yale Psychological Studies (1899), and his elaborate volume in the Yale Bicentennial Series (1901).

from the attempt to determine the density of the ether as about 10^{-17}; there are arguments (in England) just as valid for a density greater than the reciprocal of this; the whole subject is too far from settled to be given an apparently final conclusion in a text like this.

Much as we might desire to carry our review of this second volume along with the detail granted to the first, a detail which the work fully merits, we must refrain on account of the lengths to which it would lead. Moreover, the author has not been so often nor so vitally interrupted in his exposition by mathematical additions of the editors, and the text therefore appears so much more natural and homogeneous that comment is not so necessary. Finally, this volume on radiant energy and optics cannot readily and safely be judged apart from the subsequent presentation of electromagnetic theory in the fourth volume which we have not yet seen in completed form. We shall therefore run on rather rapidly, stopping merely to point out the general course of the development.

The start is strictly physical, with its description of the ether, of electric discharges and waves, of luminescence, and of radiation from hot bodies. Stefan's law, coefficients of absorption and emission, Kirchoff's law, Wien's law, Planck's formula, and the pressure of light are then treated. In dealing with the velocity of light the difference between wave velocity and group velocity is mentioned, as it should be. There is then a transition to topics in geometrical optics, reflexion, refraction, and the theory of mirrors and lenses. The facts about the index of refraction, including the case of metals, are presented with reference to various formulas which are stated without derivation. The subject is logically continued into dispersion and a lengthy discussion of spectrum analysis. There is a note by de Gramont on modern methods of spectrum observation.

After treating the transformation of radiant energy (fluorescence, phosphorescence, photography) and the measure of radiant energy (photometry, actinometry), the author goes over to optical instruments (microscope, telescope, and the like), the elements of physiological optics, and atmospheric optics. Not until then does he come to interference, and later to diffraction. These subjects, however, are covered in detail in some 120 pages. There is nothing of mathematical theory except a short note by the translator on retarded

potentials and their application to problems in diffraction. In setting forth the phenomena connected with polarization the usual rather numerous trigonometric formulas are given without any complicated mathematical analysis. The translator, however, adds a note on Green's theory of reflexion and refraction based on the notion of an elastic medium and following Lord Kelvin's presentation in his Baltimore lectures. In this connection there is a short section on the gyrostatic theory of light.

The various sorts of elastic solid theory of light which have been developed from Green to Kelvin, have undoubtedly had very eminent adherents, have still a prominent place in optical theories of the more mathematical kind, and have certainly exercised a great influence over the development of optics. Their historical prominence is therefore such that they cannot be ignored. But it is difficult to repress the feeling that they have little place in the present work. And this for at least two reasons. First they were pretty thoroughly demolished by Gibbs's comparison of Kelvin's work with the electromagnetic theory. Second they do not accord with the strictly modern, maxwellian, point of view of the author. It may be that this discord will be remedied in the fourth volume. We should not have mentioned the matter were it not for the rather elaborate extension of old fashioned theories in the following treatment of crystalline optics. It may not be amiss further to state that from an elementary, though rather abstract, point of view it would have been better at this point to have founded the theory of light upon Gibbs's almost kinematic discussion of a wave motion in a fine grained medium. This too would have been somewhat nearer to the method of Boussinesq who, though generally ranked among the elastic solid theorists, is really very careful to indicate the ways in which the ether fails to act as a solid and to avail himself of general physical reasoning more or less independent of elastic theories.

The author's final three chapters on double refraction, interference of polarized light, and rotation of the plane of polarization, covering about 200 pages, are like most of the volume chiefly descriptive. Such subjects as accidental anisotropy and liquid crystals are not forgotten or slighted. The wonder is how Chwolson could so thoroughly and acceptably have collected and arranged and set forth such a vast variety of

physical phenomena and theories as appear in the first half of his work. We must reserve the second half for notice at another time.

The volume should close here, but does not. For there is a note of 225 pages of the most mathematical character by the two Cosserats on the theory of deformable bodies. Even more than their note in the first volume is this wholly out of place, a worse than vermiform appendix. If the substance of these notes were unimportant, we could ignore them; but as they contain material which forms a distinct contribution to theoretical analytic mechanics, we shall try to give our readers an idea of their content at an early date.

From the numerous adverse comments which we have passed upon Chwolson's two volumes it might appear that we were not pleased with them. This is not so. We have found all the fault we could (a great deal of it with the editors' additions) in the hope that in future editions our criticisms might perhaps be productive of improvements. But without any changes the work is a masterpiece, and one that we should be glad to see not only in physical libraries but upon the shelves of our mathematical seminar rooms, to the end that our students of mathematics might readily avail themselves of its interesting and readable and broadening contents.

EDWIN B. WILSON.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON, MASS.

ARITHMETIQUE GENERALE.

Je trouve dans le BULLETIN (Mars, 1912) un compte-rendu de mon livre Arithmétique générale.

Je crois nécessaire et de mon droit de rectifier une grave erreur du critique. Mon arithmétique n'est pas du tout un corollaire de la physique. J'ai dit au contraire que la mathématique pure doit être à mon avis une préface de la physique. Cela veut-il dire que la mathématique doive être une science expérimentale? Nullement. Or le critique donne à entendre que ce soit là ma façon de penser. Il dit que je démontre mes théorèmes fondamentaux en me servant de "remarques faites expérimentalement sur la nature des grandeurs" et le lecteur de ce compte-rendu doit s'imaginer

que ces grandeurs sont des grandeurs physiques, ou naturelles. Cela est absolument faux.

Le critique a lu ma préface, mais il n'a pas lu le livre. Sans quoi il n'aurait pas traité ma théorie de "naïveté." Je suis aussi rigoriste que n'importe qui, et je serais le premier a traîter, non de naif, mais de farceur, celui qui prétendrait baser la mathématique, science de raison, directement sur la physique, science expérimentale.

En réalité j'ai pris pour base de mon Arithmétique générale, le rapport de deux grandeurs géométriques de même espèce: deux segments de droite, deux angles, deux fuseaux d'une même sphère, deux parallelépipèdes rectangles de même base, etc. (voir à ce sujet, ce que j'ai dit au No. 3, page 2).

J'ai reconnu, depuis la publication de mon livre, que je n'ai fait que reprendre d'une façon plus complète la base posée par Duhamel.* Celui-ci appelle rapport de deux grandeurs, leur manière d'être relative. C'est exactement la même idée que celle que j'ai développée. Mon arithmétique est donc basée sur la géométrie euclidienne, et non sur la physique. Il est vraiment étonnant que ce qu'on appelle "l'admirable théorie des proportions d'Euclide, la précision et la netteté remarquables de Duhamel, et l'admirable théorie des quaternions de Hamilton, deviennent des naïvetés quand elles sont exposées par Dumont. (Voir à ce sujet, l'Encyclopédie des sciences mathématiques, tome I, volume I, fascicule 1, pages 134 et 135).

M. Lennes n'a pas lu mon livre. Il est resté hypnotisé devant une réflexion que j'ai émise dans ma préface à propos de la définition d'une somme géométrique de vecteurs. J'ai dit que cette définition n'a pas été choisie au hasard, ni afin de satisfaire au principe de permanence de Hankel, mais bien afin de pouvoir être utilisée en mécanique pour la composition des forces, des vitesses, etc.

Cette réflexion m'avait paru utile à dire, parce que j'avais lu peu auparavant un ouvrage de M. Dassen: "Des quantités mathématiques" où cet auteur explique que si l'on a choisi la définition de la somme géométrique des vecteurs telle que chacun la connait, c'est afin de pouvoir intervertir les termes, comme dans toute somme: $\overline{AB} + \overline{CD} = \overline{CD} + \overline{AB}$. Eh bien, en voilà une de naïveté! Car si l'on veut jeter un coup

* Des méthodes dans les sciences de raisonnement, pp. 62–75.

d'oeil sur la théorie des quaternions, on y rencontrera une somme géométrique d'angles (ou d'arcs de grand cercle sur une sphère), somme qui n'est pas commutative, et qui est l'angle d'un produit de quaternions.

Et c'est cette malheureuse réflexion où j'ai parlé de forces, qui a faussé toute la compréhension que M. Lennes s'est faite de ma théorie.

Il n'a pas vu que j'ai tout simplement adapté à tous les nombres, nombres absolus, nombres qualifiés et nombres complexes, les définitions qu'Hamilton avait créées pour les quaternions.

Lorsque j'ai parlé d'expérience, j'ai également été mal compris. Il ne s'agit d'expériences à faire sur des grandeurs physiques, il s'agit de l'expérience, de la connaissance que le penseur a du monde qui l'environne. Or la géométrie est précisément l'image idéale de ce monde. L'homme l'a créée afin de pouvoir plus facilement étudier l'univers. Et c'est donc la géométrie, science rigoureuse, que j'ai prise comme base essentielle de mon Arithmétique, afin que l'arithmétique concours immédiatement, comme la géométrie, à l'étude de l'univers.

Les théories arithmétiques des logiciens ne peuvent satisfaire un esprit hanté par le problème de la rotation de la terre dans le champ magnétique du soleil. Ces théories elles-mêmes qui se piquent d'être pures de tout compromis avec les notions expérimentales (physiques) poursuivent une utopie et se bercent d'illusion. Voici en quoi. Elles se bercent d'illusion en ce sens que toutes ces théories reposent sur la notion d'ordre. La première définition, pierre fondamentale de tout l'édifice, est la suivante: On appelle nombres entiers, une suite de mots ou de signes se succédant dans un ordre caractéristique, arbitrairement choisi une fois pour toutes. Ces signes n'ont aucune signification par eux-mêmes. Ainsi 5 est le signe qui suit 4 et qui précède 6. Rien de plus. La suite commence au caractère 1:

$$1.\ 2.\ 3.\ 4.\ 5.\ 6.\ 7.\ \cdots.$$

Eh bien, dire que 5 vient après 4, ou suit 4, ou succède à 4, ou est à droite de 4, c'est faire appel à la notion temps! ou à la position des organes dans le corps humain (la droite et la gauche).

Les autres définitions, fraction, nombre irrationnel, nombre

qualifié, nombre complexe, c'est-à-dire, tous groupes de deux nombres écrits dans un ordre caractéristique, font également appel à cette notion d'ordre, c'est à dire, de temps. C'est, je pense, ce qui avait fait dire à Gauss que l'arithmétique est la science du pur temps.

C'est enfin une utopie que de prétendre émettre des pensées débarrassées des concepts fondamentaux que le monde où nous vivons a mis dans notre conscience; pensées qui n'en sont que le produit, synthétisé grâce à notre mémoire consciente ou non.

Pour en revenir à ma méthode, je dirai que je suis aussi désireux que n'importe qui, de découvrir où se cachent des trous dans mes démonstrations, et le cas échéant, comment il faudrait les combler. Malheureusement les critiques n'ont pas le temps de lire consciencieusement les livres qu'ils doivent analyser. Il en est en mathématique comme dans les salons bien fréquentés: Chacun ne s'intéresse à la conversation que pour la part qu'il y prend.

J'espère que vous voudrez bien insérer cette réponse à votre compte-rendu, et que M. Lennes reconnaîtra de bonne foi qu'il c'est trompé quant à la valeur scientifique de mon point de départ.

E. DUMONT.

I do not deem it necessary to refer in detail to the strictures which Professor Dumont has made on my recent review of his Arithmétique générale. I need only to repeat that there are no explicitly stated postulates in the book, that what are usually regarded as such in arithmetic are proved as theorems by general reference to statements about magnitudes in general. These statements about magnitudes in general are obtained quasi-inductively from geometric considerations. In this sense the book is truly naïve, and in this sense Hamilton's treatment may well be regarded as naïve.

N. J. LENNES.

SHORTER NOTICES.

The Theory of Determinants in the Historical Order of Development. Volume II: *The period* 1841 *to* 1860. By THOMAS MUIR, Superintendent-General of Education in Cape Colony. London, Macmillan and Company, 1911. xvi + 475 pp.

THE general arrangement of the present volume is similar to that of volume I, which appeared in 1906 and was reviewed in the BULLETIN, volume 13 (1907), page 244. It is composed, mainly, of a list of books and articles which appeared during the twenty years from 1841 to 1860, and of brief remarks on their contents. A chapter is assigned to each of the following thirteen special forms: axisymmetric determinants, alternants, compound determinants, recurrents, Wronskians, Jacobians, skew determinants, orthogonants, persymmetric determinants, bigradients, Hessians, circulants, and continuants. The last chapter deals with less common forms,—including permanents, determinants with complex elements, determinants connected with anharmonic ratios, and Sylvester's unisignant.

The first two chapters of the book are devoted to determinants in general, and cover 108 pages. In the chapter on continuants the author has gone beyond the limits to which he confined himself in the other parts, by pursuing this subject up to 1870. Articles and books which relate to different forms are considered under each of the forms with which they have direct contact, and hence the same work is often listed and considered in a number of different chapters. The book closes with a list of the 87 different authors "whose writings are reported on," but there is no subject index. Such an index would have been especially useful in view of the large number of different terms employed by the various authors whose writings are considered.

It was more than thirty years ago that Muir began the collection of the material which has served as a basis for the present work, and his extensive lists of writings on determinants, which have appeared in the *Quarterly Journal of Mathematics*, volumes 18, 21, and 36, are well known. Hence one does not expect to find many omissions in a work relating to the comparatively short period considered in the present

volume. It seems, however, that a history of determinants during this period should not fail to mention Weierstrass—especially his article which appeared in 1858 in the Berliner *Berichte*.* In the history of Hermitian determinants (page 449) reference should also have been made to the articles by Hermite which appeared in volume 47, page 345, and in volume 53, page 183, of *Crelle*.

What appears to the reviewer as the most serious defect in the present work results from the fact that the chronological arrangement of the articles is frequently based solely on the dates printed at the end of the articles, without any reference to the date of publication. For instance, Spottiswoode's memoir entitled "Elementary theorems relating to determinants, second edition," which appeared in *Crelle* in 1856, is classed with the papers which appeared in 1853. The unreliability of the dates at the end of articles was discussed by Eneström in the *Bibliotheca Mathematica*, volume 5 (1904), page 196; and, in this instance, Eneström seems to agree with Cantor's dictum, "da die Geschichte unwiderruflich die Veröffentlichungszeit als allein massgebend betrachten muss, wo Erstlingsrechte zu vergeben sind . . ."† It seems to the reviewer that there is a growing tendency to omit dates at the end of articles and he believes that this is desirable.

Among the few minor errors which the reviewer has noticed is one on page 206 relating to the date of an article by Bazin, entitled "Sur une question relative aux déterminants," which appeared in *Liouville*, volume 16 (1851). This article is classed in the present volume with those of 1854 and the date assigned to it is April, 1854. The number of minor errors seems, however, small; and, while the reviewer does not consider the present volume to be as complete and useful as volume I, yet it must be regarded as an important addition to the literature on determinants, and it involves a large amount of interesting information in a fairly convenient form. The fact that its author is continuing his mathematical work notwithstanding his administrative duties is an element of inspiration.

G. A. MILLER.

* Cf. Pascal's Repertorium, vol. 1 (1910), p. 66.

† Cantor, Vorlesungen über Geschichte der Mathematik, second edition, vol. 2 (1900), p. 811.

An Introduction to the Lie Theory of One-Parameter Groups with Applications to the Solution of Differential Equations. By Dr. ABRAHAM COHEN. D. C. Heath and Company, 1911. vii + 248 pp.

THE theory of continuous groups has become a subject of such general importance, and constitutes a method of such great power, that every attempt to render it more easily accessible should be received gratefully. One of the advantages of the Lie theory is its elementary character, in so far as a large portion of it may be developed with very modest prerequisites. So much the more must we welcome a book which enables us to present this subject to undergraduate classes in our own language. It is always a serious thing to complicate matters for the younger students by adding linguistic to their inevitable mathematical difficulties. The large works of Engel and Scheffers on Lie's theory are written with admirable pedagogic insight, so that there would be little need of such a book as this in German. But the only larger English book on the subject, that by Campbell, is far from being a paragon of lucid exposition. The little book by Page is hardly extensive enough to serve the purpose of more than a mere introduction.

There is then a real field for such a book as Dr. Cohen's. We feel, however, as though the author had overemphasized the differential equation part of the theory. Still, that is a matter of taste.

The book is divided into seven chapters and an appendix. The first chapter naturally begins with the definition of a group of transformations, infinitesimal transformations, invariants, path curves, canonical variables, etc. In Chapter II these notions are applied to differential equations of the first order in connection with the theory of the integrating factor. The third chapter has the heading Miscellaneous theorems and geometrical applications, and deals among other things with isothermal curves. Chapter IV is on differential equations of the second and higher orders, and introduces the notion of n-times extended group. Chapter V is devoted to linear partial differential equations of the first order, while Chapter VI returns to ordinary differential equations of the second order. Chapter VII finally is on contact transformations. In the appendix are developed a number of the more difficult parts of the general theory, these matters being referred to in the

proper place in the main body of the text. Besides, the appendix contains two tables to serve as an aid in properly classifying and integrating differential equations.

The book is well written and is well adapted as a general introduction to Lie's theory. As already indicated, the too systematic treatment of differential equations from this point of view appears to the reviewer to be a defect. It is one thing to recognize that Lie's theory is capable of bringing some order into the haphazard methods of the elementary theory of differential equations; but is is quite another to use Lie's theory as a strait jacket which every differential equation must be made to fit.

And now for a few points of detail. The author's definition of a one-parameter group is rather clumsy, as a result of the unfortunately so common desire to put it all into one sentence. The two parts of remark 1 on page 7 are not obviously connected. The true reason for the validity of remark 1 is to be found in the latter part of remark 2. Although the author speaks everywhere of one-parameter groups, he really means such groups as are generated by an infinitesimal transformation. At least, a number of his theorems are not true for mixed groups. On page 176 in deriving the condition for a "union of lineal elements" he neglects the case where the union consists of the elements of a fixed point. The printing is well done. The reviewer has noticed only a few misprints. On page 45 last line read $-A\eta$ instead of $+A\eta$. Page 177 in Ex. 5 read $-\cot t$ instead of $-\cos t$. Page 188 just above equation (130′) read characteristic instead of charactertisic.

A rather confusing feature is the use of two different scarcely distinguishable fonts for the same letters, as on page 24. There is no necessity for this. It seems a pity that the publishers have not seen fit to give the book a wider margin. The binding also is none too good.

E. J. WILCZYNSKI.

Abriss der Algebra der Logik. Bearbeitet im Auftrag der Deutschen Mathematiker-Vereinigung von Dr. EUGEN MÜLLER, Professor an der Oberrealschule zu Konstanz. In drei Teilen. Erster und Zweiter Teil. Leipzig und Berlin, B. G. Teubner, 1909–1910, 159 pp. 8 marks.

ALMOST from the days of Boole an ever-increasing need has been felt for an elementary introduction to the growing subject

of the algebra of logic. There have been numerous extended and more or less exhaustive treatises on the subject, notably that of Dr. Ernst Schröder, but few of an elementary character.* The present work has the name of Dr. Schröder at the top of title page and was inspired by the references in several places in Schröder's Vorlesung to a proposed "Abriss." The first part, which serves as an introduction, was partly prepared from an outline and manuscript fragments left by Schröder. The first twenty pages are given up to the elementary introduction to the ideas, terms, and symbols used. The standpoint used here is that of the algebra of propositions or implications. This part of the book is rather diffuse and the entire work contains much that is obvious, and could be condensed with advantage. This is probably intended to appeal to the non-mathematical or philosophical reader. The seven axioms on which the subject is based are given on pages twenty and twenty-one. This part is interpreted from the standpoint of the algebra of inclusion or classes or of incident regions, and the two phases are treated from this point on in parallel.

The axioms are as follows:

I. $a \nleq a$, Identitäts- oder Tautologieaxiom.
II. $(a \nleq b)(b \nleq c) \nleq (a \nleq c)$, Subsumtionsschluss.
III. $(a \nleq b)(b \nleq a) = (a = b)$, Gleichheitsdefinition.
IV×. $0 \nleq a$, Nullpostulat. IV+. $a \nleq 1$, Einspostulat.
V. $1 \nleq 0$ Existenzpostulat.
VI×. $(x \nleq a)(x \nleq b) = (x \nleq ab)$, Produktdefinition.
VI+. $(a \nleq y)(b \nleq y) = (a + b \nleq y)$, Summendefinition.
VII × +. $(a + z)(\bar{a} + z) = z = az + \bar{a}z$, Negations- oder Distributionsprinzip.

These axioms are explained and some alternate axioms are given in the next five pages. The rest of part one (to page 50) is taken up with the deduction of theorems which are the immediate consequences of these axioms. Most of the relations in part one are expressed in terms of the asymetrical sign of implication or inclusion under the general term of "subsumption" ($\nleq$), and the sign of equality is defined by Axiom III in terms of the sign of subsumption. There is a logical fallacy involved here in using the equality sign in its

* Except the work by Couturat I know of no other of a general character.

own definition.* A better definition would be composed of the two parts†

$$(a = b) \notin (a \notin b)(b \notin a), \quad \text{and} \quad (a \notin b)(b \notin a) \notin (a = b).$$

The equality sign is used however before Axiom III is given. In part two, Aussagentheorie, Funktionen, Gleichen und Ungleichen, the author derives and discusses at length the general formulas for the symbolic sum and product, and the methods of substitutions and reduction of symbolical functions with the methods of elimination and solution of the general symbolical equation. This part is quite extensive and there is much that is obvious.

One good feature of the work, at least from the standpoint of the mathematician, is the numerical or algebraical examples, illustrative of the general theory, used throughout the book. The Abriss on the whole is a very creditable piece of work and cannot but help arouse interest in the algebra of logic. No serious errors occur, but there are some obvious misprints of the type found in line three, page thirteen, where a in the last parenthesis should be α.

L. I. NEIKIRK.

Cours d'Astronomie. Première partie: *Astronomie théorique.* Par H. ANDOYER. Deuxième édition entièrement refondue. Paris, Librairie scientifique A. Hermann. 383 pp.

THE first edition of this work in hectographed form has been reviewed in volume 13, number 10, of the BULLETIN.‡ The second edition is much enlarged, almost doubled in size. The number of chapters is increased from fifteen to nineteen. The work in its present form is subdivided into four large sections, or books, of which the first is of a more analytical character. The transformation of coordinates with common origin—spherical trigonometry—is given with considerable completeness; to it is added the transformation for different origins and parallel systems of axes—the analytical basis for parallax and aberration. The second book defines the adopted systems of coordinates in astronomy, the concepts of sidereal and solar

*A similar fallacy is evident in axiom VI×. It would be difficult to avoid this in a simple way.

†A combination of his III′, III″ and III‴.

‡See also the review of the second part, by Professor Longley, BULLETIN, vol. 15, pp. 467–468.

time, and gives, as an application of the theories of the first book, the theories of parallax and aberration themselves. This book contains likewise some fundamental notions about geodesy, which is in proper bearing with the greater extent with which the theory of spherical trigonometry has been given in this work. That the theory of refraction is likewise included in this second section is of course due to the fact that the correction for refraction comes, in the order of application, even before those for parallax and aberration. The third book opens with a rather complete study of certain fundamental notions of celestial mechanics which are made use of in the theory of precession and nutation and of time, all of which theories are set forth in this section. The geocentric motions of planets and satellites, including the moon, complete the third book; their inclusion, in the text before us, is much to be commended, since the older textbooks on this subject give almost no attention to them. Inasmuch as observations of eclipses of moon and sun and occultations of stars define certain directions in space, to which the usual corrections are to be applied, the last section of the work is devoted to their theories. Moreover the theory of transformation of co-ordinates finds in the prediction of the times of eclipses an extensive field of application.

The name of the author and the reputation of the publishing house are from the start sufficient guarantees for the excellent character of the book. As far as has been possible to ascertain, typographical errors of any importance seem to be happily avoided; that the types for the fractional lines have not always been perfect is perhaps the only criticism one can make of the typographical appearance of the book. The introduction of a general index and copious references at the end of chapters would have been a distinctive improvement; as it stands, the beginner is likely to miss the historical setting, and permit himself to be carried too much by the "ipse dixit" of the author. It is believed that the mathematician, interested in the subject, will find Professor Andoyer's book extremely attractive, since everywhere the underlying mathematical principles are strongly brought into the foregound. His attention is especially called to the chapter on refraction, in which Radau's presentation is given in preference to those of Laplace, Bessel, or Ivory. A simple reference to Bruhns's Atmosphärische Strahlenbrechung would have supplied the

reader with a pretty complete literature of this highly interesting subject.

The reviewer would have liked to see a chapter on interpolation and related subjects form part of the book, since an intelligent use of the Ephemerides is not to be thought of without a mastery of this topic. The astrophysicist will be somewhat disappointed by not seeing the transformation of velocities included in the third chapter of the book. The student interested in the photometric measurements of the satellites of Jupiter after Pickering and Sampson might well claim that the theory of the photometric eclipses is surely as important as the passages of Mercury and Venus over the disc of the sun, to which the author has devoted 8 pages. Finally in the subject of differential refraction a recognition of the peculiar formulas for measurement of photographic plates might not have been entirely out of place.

KURT LAVES.

NOTES.

AT the meeting of the Edinburgh mathematical society on May 10 the following papers were read: By W. P. Milne, "Investigations on circular cubics and bicircular quartics"; by G. Teixeira, "Notice sur les recherches de Maclaurin concernant les cubiques circulaires"; by H. T. Pioggio, "Note on linear differential equations with constant coefficients."

THE annual meeting of the Deutsche Mathematiker-Vereinigung will be held in affiliation with the eighty-fourth convention of German naturalists and physicians at Münster in Westphalia September 15–21 under the presidency of Professor W. v. DYCK. While papers in other subjects will be welcome, particular emphasis will be put upon reports and new contributions to the theory of differential geometry. Information regarding the meeting can be obtained from the secretary, Professor A. Krazer, Karlsruhe, Westendstrasse 57.

THE Macmillan Company announce the publication during the summer of a work on the calculus by Professors E. W. DAVIS and W. C. BRENKE, of the University of Nebraska.

THE second volume of Professor J. PIERPONT'S Theory of functions of a real variable is in press and will be published by Ginn and Company in July.

HENRY Holt and Company announce as in preparation a work on analytic geometry of space by Professors Virgil SNYDER and C. H. SISAM.

UNIVERSITY OF MICHIGAN. The following advanced courses in mathematics are announced for the academic year 1912–1913: By Professor W. W. BEMAN: Advanced calculus, two hours; Higher plane curves, two hours, second half-year; Linear differential equations, two hours, second half-year.—By Professor ALEXANDER ZIWET: Theory of potential, three hours, first half-year; Hydrodynamics, two hours; Vector analysis, three hours, second half-year; Integral equations, two hours, second half-year.—By Professor J. L. MARKLEY: Theory of functions I, three hours; Theory of functions II, two hours.—By Professor J. W. GLOVER: Seminary in probabilities, two hours.—By Professor W. B. FORD: Harmonic analysis, two hours; Infinite series and products I, two hours; Infinite series and products II, two hours.—By Professor PETER FIELD: Theory of twisted curves and surfaces, two hours; Advanced mechanics, three hours.

PROFESSOR J. HADAMARD has been appointed professor of mathematical analysis at the Ecole polytechnique of Paris, as successor to Professor CAMILLE JORDAN, retired.

THE Göttingen academy has awarded M5,000 from the Wolfskehl foundation to Professor E. ZERMELO, of the University of Zurich, for his researches in the theory of sets.

DR. L. SCHRUTKA, of the technical school of Vienna, has been appointed associate professor of mathematics at the German technical school at Brünn.

THE University of Colorado has conferred the honorary degree of doctor of laws on Professor FLORIAN CAJORI, of Colorado College.

PROFESSOR VIRGIL SNYDER, of Cornell University, has been granted leave of absence during the first half of the academic year 1912–1913, which he will spend in travel abroad.

AT the University of Minnesota, Professor H. T. EDDY, head of the department of mathematics and mechanics of the college of engineering and dean of the graduate school, has

retired from active service. Professor W. E. BROOKE has been appointed to succeed Professor Eddy. Dr. W. F. HOLMAN has been appointed to a professorship of engineering mathematics, to succeed Professor A. E. Haynes, retired.

CATALOGUES of second-hand books: Henry Sotheran and Co., 140 Strand, London, catalogue 725, mathematics and allied subjects, about 2,000 titles.—G. E. Stechert and Co., 151 West 25th Street, New York, scientific journals and proceedings, about 700 titles.—Emile Blanchard, 10 rue de la Sorbonne, Paris, catalogue 65, 1,167 titles in pure and applied mathematics.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

BERGHOLZ (O. A.). Die Lösung des Fermatschen Problems $x^n + y^n = z^n$. Dessau, Art'l, 1912. 8vo. 19 pp. M. 1.00

BLENCK (G.). Untersuchungen über das Amiotsche Theorem bei den Flächen zweiter Ordnung und über Erzeugungsarten des elliptischen Kegels. (Diss.) Rostock, 1911. 8vo. 91 pp.

BLUHM (B.). Ueber konjugierte Kurven und Flächen. (Diss.) Königsberg, 1911. 8vo. 89 pp.

BÖGER (R.). Symmetrische Involutionen und fokal getrennte Geraden. (Progr.) Hamburg, 1912. 8vo. 56 pp.

BRINCARD (R.). See DUPORCQ (E.).

BURALI-FORTI (C.). Corso di geometria analitico-proiettiva. Torino, Gallizio, 1912. 8vo. 7+268 pp. Cloth. L. 8.00

CARSLAW (H. S.). An introduction to the infinitesimal calculus. 2nd edition. London, Longmans. 8vo. 4s.

CHEMNITZER (B.). Die kürzesten Linien auf der Kreisringfläche. 1ter Teil. (Progr.) Chemnitz, 1912. 4to. 19 pp.

CREFCOEUR (A. J. M.). Cours d'analyse infinitésimale. 3e partie: Calcul des probabilités ou théorie analytique du hasard. 2e édition, revue et augmentée. Liège, Béranger, 1911. 8vo. 253 pp. F. 7.00

DEDEKIND (R.). Stetigkeit und irrationale Zahlen. 4te, unveränderte Auflage. Braunschweig, Vieweg, 1912. 8vo. 7+24 pp. M. 1.00

——. Was sind und was sollen die Zahlen? 3te, unveränderte Auflage. Braunschweig, Vieweg, 1912. 8vo. 21+58 pp. M. 1.80

DUPORCQ (E.). Premiers principes de géométrie moderne. 2e édition, revue et augmentée par R. Brincard. Paris, Gauthier-Villars, 1912. 8vo. 7+175 pp.

ECKELHARDT (F.). Differential- und Integralrechnung. Wien, Pichler, 1912. M. 0.85

ENCYCLOPÉDIE des sciences mathématiques pures et appliquées. Edition française. Tome I. 2e volume: Algèbre. Fascicule 4: Théorie des formes et des invariants, par F. W. Meyer et J. Drach. Leipzig, Teubner, 1912. 8vo. Pp. 425–520. M. 3.60

FESTSCHRIFT. Heinrich Weber zu seinem 70. Geburtstag am 5. März, 1912. Gewidmet von Freunden und Schülern. Mit einem Bildnis von H. Weber. Leipzig, Teubner, 1912. M. 24.00

FUNK (P.). Ueber Flächen mit lauter geschlossenen geodätischen Linien. (Diss.) Göttingen, 1911. 8vo. 22 pp.

GYR (W. X.). Die Polaren der Lemniskate. (Diss.) Bern, 1911. 8vo. 59 pp.

HEFFTER (L.). Ueber eine vierdimensionale Welt. (Öffentliche Antrittsrede.) Freiburg, Speyer und Kaerner, 1912. 8vo. 26 pp. M. 0.90

HERMITE (C.). Oeuvres, publiées sous les auspices de l'Académie des sciences, par E. Picard. Tome III. Paris, Gauthier-Villars, 1912. 8vo. 528 pp. Fr. 18.00

KNESER (A.). Mathematik und Natur. (Rektoratsrede.) Breslau, Trewendt und Granier, 1912. 8vo. 14 pp. M. 0.50

LEMESSURIER (T. A.). Key to W. W. Johnson's Differential Equations. New York, Wiley, 1912. 8vo. 5+125 pp. $1.75

MACNEISH (H. F.). Linear polars of the k-hedron in n-space. (Diss.) Chicago, University of Chicago, 1912. 8vo. 32 pp. $0.25

NEUMANN (R.). Die Entwicklung willkürlicher Funktionen nach den Hermiteschen und Laguerreschen Orthogonalfunktionen auf Grund der Theorie der Integralgleichungen. (Diss.) Breslau, 1912. 8vo. 111 pp.

NOTARBARTOLO (G.). Metodo rapido con semplici addizioni per la costruzione di tavole delle potenze di numeri della serie naturale. Acqui, Tirelli, 1912. 8vo. 14 pp.

——. Valore costante delle differenze dell' n° ordine, ecc. Torino, Unione tipografico-editrice, 1912. 4to. 10 pp.

PICARD (E.). See HERMITE (C.).

REIDER (R.). Polynomische Entwicklungen von Funktionen einer komplexen Variabeln. (Diss.) Basel, 1911. 8vo. 85 pp.

ROSE (M.). Einleitung in die Funktionentheorie (Theorie der komplexen Zahlenreihen). (Sammlung Göschen, Nr. 581.) Leipzig, Göschen, 1912. 8vo. 139 pp. Cloth. M. 0.80

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WIENER (F. W.). Elementare Beiträge zur neueren Funktionentheorie. (Diss.) Göttingen, 1911. 8vo. 40 pp.

WILSON (E. B.). Advanced calculus. A text upon select parts of differential calculus, differential equations, integral calculus, theory of functions. Complete edition. Boston, Ginn, 1912. 8vo. 9+566 pp. $5.00

ZECHSTEIN (P. G.). Die Regel des Pythagoras und die Behauptung des Fermat. Berlin, Buschhardt, 1912. 8vo. 4 pp. M. 2.00

II. ELEMENTARY MATHEMATICS.

BATES (E. L.) and CHARLESWORTH (F.). Practical mathematics and geometry. Part 2: Advanced course. New York, Van Nostrand, 1912. 12mo. 776 pp. $1.50

BÔCHER (M.). Graduate work in mathematics in universities and in other institutions of like grade in the United States. (International commission on the teaching of mathematics.) Washington, 1911.

BOURLET (C.). Algèbre. 1re, 2e et 3e années. Paris, Hachette, 1912. 16mo. 228 pp. Cartonné. Fr. 2.00

BÜRKLEN (O. T.). Aufgabensammlung zur analytischen Geometrie der Ebene. 2te, verbesserte Auflage. (Sammlung Göschen, Nr. 256.) Leipzig, Göschen, 1912. 8vo. 175 pp. Cloth. M. 0.80

CHARLESWORTH (F.). See BATES (E. L.).

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DALLE (A.). 2000 théorèmes et problèmes de géométrie avec solutions. Namur, Wesmael-Charlier, 1912. 8vo. 8+825 pp. Fr. 15.00

—— et DE WAELE (C.). Cours de géométrie. Geometrie plane. 3e édition. Namur, Wesmael-Charlier, 1912. 8vo. 328 pp. Fr. 2.75

FAIFOFER (A.). Elementi di geometria. 17a edizione. Venezia, Sorteni e Vidotti, 1911. 8vo. 533 pp. L. 4.00

—— Il libro di geometria. 4a edizione. Venezia, Sorteni e Vidotti, 1911. 8vo. 153 pp. L. 1.50

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HALL (A. G.) and FRINK (F. G.). Trigonometric and logarithmic tables. New York, Holt, 1911. 8vo. 3+97 pp. $0.75

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——. A school algebra. Part 3. With or without answers. London, Macmillan. 8vo. 1s. 6d.

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KUTNEWSKY (M.). See MÜLLER (H.).

LIETZMANN (W.) und SCHIMMACK (R.). Berichte, veranlasst durch die Internationale Unterrichtskommission. Heft 7: Der Kongress in Mailand vom 18. bis 20. September, 1911, von W. Lietzmann. Ueber die Verschmelzung verschiedener Zweige des mathematischen Unterrichts. Leipzig, Teubner, 1912. M. 1.60

MEHLER (F. G.). Elementarmathematik. Bearbeitet von A. Schulte-Tigges. Ausgabe B. 1ter Teil: Synthetische Geometrie der Kegelschnitte. 2te, unveränderte Auflage. Berlin, Reimer, 1912. 8vo. 9+72 pp. M. 1.50

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—— und KUTNEWSKY (M.). Aufgaben aus der Arithmetik, Trigonometrie und Stereometrie. 2ter Teil. Ausgabe B. 4te Auflage. Leipzig, Teubner, 1912. 8vo. 12+320 pp. Cloth. M. 3.00

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SERRAZANETTI (G.). Elementi di algebra. Bologna, Zanichelli, 1912. 16mo. 100 pp. L. 1.25

SMOLIK und HELLER. Raumlehre und darstellende Geometrie. Bearbeitet von K. Hahndel. 4te Auflage. Wien, Tempsky, 1912. 8vo. 211 pp. Cloth. M. 3.60

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WELPTON (W. B.). See Moss (C. E.).

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FEURSTEIN (K.). Elastischer Zustand und Spannungsverteilung im Erdinneren. (Diss.) Freiburg, 1912. 8vo. 70 pp.

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